TABLE OF CONTENTS

GATE - Civil Engineering

Unit - 1 ENGINEERING MECHANICS

SYLLABUS

Engineering Mechanics: System of forces, free-body diagrams, equilibrium equations; Internal forces in structures; Frictions and its applications; Centre of mass; Free Vibrations of undamped SDOF system.

CONTENTS

CHAPTER – 1

FORCES & MOMENT SYSTEM

1. Introduction

- A branch of physical science that deals with the state of rest or motion.

  Three broad classification are

  1. Classical / Newtonian mechanics

  2. Relativistic mechanics

  3. Quantum/ Wave mechanics

1.1 Classical mechanics /Newtonian Mechanics

- Developed by "Sir Isaac Newton".
- Mechanics of bodies based on three laws of motion & law of gravitation is called Newtonian mechanics or classical mechanics.

1.2 Relativistic Mechanics

Developed by "Albert Einstein.

- Mechanics used to explain behaviour of high-speed bodies (speed of light = 3×10^8 m/s) is called relativistic mechanics.

1.3 Quantum mechanics

- Developed by Schrodinger and Broglie.
- Mechanics used to explain behavior of particles when atomic distances are considered is called quantum / wave mechanics.

2. Engineering Mechanics

- Application of laws of mechanics to field problem is known as engineering mechanics.

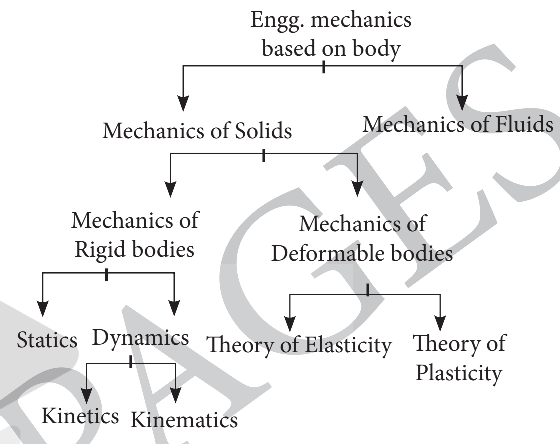


Fig 1: Classification of Mechanics

2.1 Statics

- Mechanics of rigid body which deals with forces and its effect on body at rest or in equilibrium.

2.2 Dynamics

- Mechanic of rigid body which deals with forces and its effect on body in motion.

2.2.1 Kinematics

- Branch of dynamics which deals with bodies in motion without considering the forces causing motion.

2.2.2 Kinetics

- Branch of dynamics which deals with bodies in motion along with considering forces causing motion

3. Common terms related to mechanics

3.1 Matter

- Anything that occupy space and possess mass. The form of matter are solid, liquid, gas, plasma, Bose Einstein condensate, Quark gluon, Degenerate matter etc..

CHAPTER – 10
LIMIT STATE OF COLLAPSE – COMPRESSION

1. Compression Members

1.1. Column

It is a compression member whose effective length is greater than 3 times the least dimension of the member i.e., (l_{eff} /b > 3).

1.2. Pedestal

It is a compression member whose effective length is less than 3 times the least dimension of the member i.e., (l_{eff} /b ≤ 3).

1.3. Strut

The member of a truss which is under the axial compressive force is called as strut.

1.4. Boom

The member of a crane which is under compressive force is known as boom.

2. Classification Based on Slenderness Ratio

- Slenderness ratio (λ) of an RCC compression member is a ratio of effective length of the compression member to its least lateral dimension.

$$\lambda = \frac{l_{eff}}{b}$$

- If $\lambda \leq 3$ ⇒ Pedestal
- If $\lambda > 3$ ⇒ Column
- $3 < \lambda < 12$ ⇒ Short column
- $\lambda \geq 12$ ⇒ Long column

3. Minimum Eccentricity [Clause 39.2 & 25.4]

All columns shall be designed for a minimum eccentricity (e_{min})

$$e_{min} = \frac{\text{unsupported length of column}}{30} + \frac{\text{lateral dimension}}{30}$$

subject to a minimum of 20 mm.

where the calculated eccentricity is larger, the minimum eccentricity should be ignored.

4. Short Columns

4.1. Assumptions [Clause 39.1]

In addition to the assumptions made for flexure, the following shall be assumed:

4.1.1. The maximum compressive strain in concrete in axial compression is taken as 0.002.

4.1.2. The maximum compressive strain at the highly compressed extreme fibre in concrete subjected to axial compression and bending and when there is no tension on the section shall be, $0.0035 - 0.75\times(\varepsilon_C)$

where ε_C is the strain at the least compressed extreme fiber.

4.2. Design criteria for axially loaded short columns [Clause 39.3]

- Shall be designed for minimum eccentricity
- When $e_{min} \ngtr 0.05$ times the lateral dimension, members shall be designed by the following equation:

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

where P_u = Axial load on the member, A_c – Area of concrete, f_y – Characteristic strength of compression reinforcement, A_{sc} – Area of longitudinal reinforcement for column, A_g – Gross area

4.3. Short column with helical reinforcement

- For compression members with helical reinforcement the strength shall be taken as 1.05 times the strength of similar member with lateral ties (Clause 39.4).

  i.e., $P_u = 1.05[0.4f_{ck} A_c + 0.67f_y A_{sc}]$

  where A_c – Area of core of column measured to the outside diameter of helix

- It should also satisfy the condition [Clause 39.4.1],

$$\frac{\text{Volume of helical reinforcement}}{\text{Volume of core}}$$

$$0.36\left[\left(\frac{A_g}{A_c}\right)-1\right]\left(\frac{f_{ck}}{f_y}\right)$$

  where, A_g = Gross area of the section, A_c – Area of the core of the helically reinforced column measured to the outside diameter of the helix.

4.4. Design of short column subjected to uniaxial bending

The maximum strain in concrete at the outermost compression fibre is 0.0035 when the N.A lies within the section and

- In the limiting case when the N.A lies along the edge of the section, in the latter case strain carries from 0.0035 at the highly compressed edge to zero at the opposite edge.
- For purely axial compression, the strain is assumed to be uniform and equal to 0.002 across the section.

  The strain distribution lines in the above two cases intersect each other at a depth of 3D/7 from the highly compressed edge. This point is assumed to act as a fulcrum for the strain distribution line when the N.A lies outside the section.

- This leads to the assumption that the strain at the highly compressed edge is 0.0035 minus 0.75 times the strain at the least compressed edge.

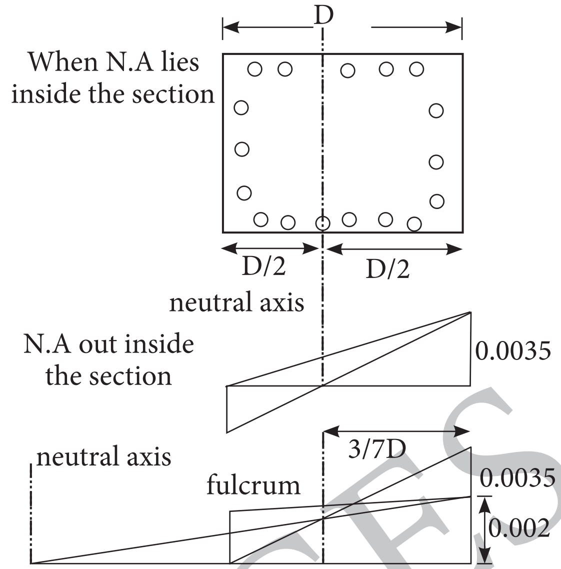


Fig. 1 Strain distribution for short column

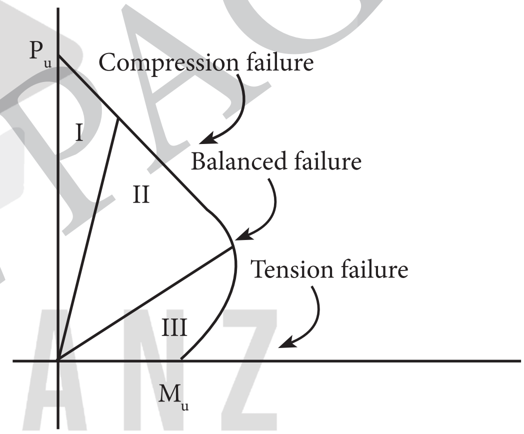


Fig. 2 Interaction curve

- The design of member subject to combined uniaxial loading and uniaxial bending will involve lengthy calculations by trial and error. In order to overcome these difficulties interaction diagrams may be used.
- They have been prepared and published by BIS in SP 16 Design aid for reinforced concrete to IS 456.

4.5. Short column with biaxial bending [Clause 39.6]

- The Load Contour Method given by Brelser 1960 is used.
- The column subjected to biaxial moment should satisfy the following equation,

$$\left(\frac{M_{ux}}{M_{ux1}}\right) + \left(\frac{M_{uy}}{M_{uy1}}\right) \ngtr 1$$

where M_{ux}, M_{uy} = Factored moments about x & y axes due to design loads, M_{ux1}, M_{uy1} = Maximum uniaxial moment capacity for an axial load of P_u, bending about X & Y axes respectively.

α_n = coefficient which depends on cross sectional dimension, the amount of reinforcement, concrete strength and yield strength of steel.

4.6. Slender compression members

- Additional moment method is applied in LSM.
- Additional moments due to slenderness of the column are given by

$$M_{ux} = \frac{PD}{2000}\left(\frac{L_{ex}}{D}\right)^2$$

$$M_{av} = \frac{Pb}{2000}\left(\frac{L_{ey}}{b}\right)^2$$

[Clause 39.7.1]

- The above moments should be added to the moments due to eccentric loads.

where L_{ex}, L_{ey} = Effective length along major and minor axes respectively, M_{ax}, M_{ay} = Additional moments about major and minor axes respectively, b & D = width & depth of the cross section (depth is at right angles to the major axis)

5. Codal Specifications

5.1. Slenderness limits for columns

- With both ends restrained, unsupported length < 60 times lateral dimension (Clause 25.3.1)
- If one end of the column is unrestrained, unsupported length $\ngtr \frac{100\,b^2}{D}$ (Clause 25.3.2)

5.2. Minimum percentage of longitudinal reinforcement = 0.8% (Clause 26.5.3.1)

5.3. Maximum reinforcement $\ngtr$ 6% (preferably 4%)

5.4. Minimum percentage of steel shall be based on actual area of concrete to resist direct stress and not upon actual area.

5.5. Minimum size of longitudinal bars= 12 mm (to avoid buckling).

5.6. Spacing of longitudinal bars $\ngtr$ 300 mm.

5.7. Minimum number of bars,

For square columns = 4

For circular columns = 6

For hexagon = 6 (one at each corner)

5.8. Minimum percentage of reinforcement for pedestal = 0.15% of gross cross sectional area (nominal).

5.9. Pitch & diameter of lateral ties [Clause 26.5.3.2 (c)]

Pitch not more than
- Least lateral dimension of the compression member
- 16 x minimum diameter of longitudinal reinforcement
- 300 mm

Diameter $\nless$
- ¼th of diameter of largest longitudinal bar
- 6 mm

5.10. Helical reinforcement [Clause 26.5.3.2(d)]

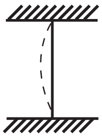
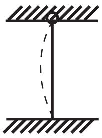
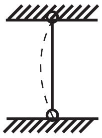
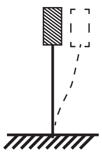
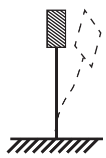
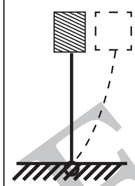
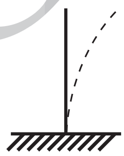
Pitch $\ngtr$
- 75 mm
- 1/6th core diameter of column

Pitch $\nless$
- 25 mm
- 3 × diameter of steel bar forming the helix

5.11. The clear cover to longitudinal reinforcements $\nless$ 40 mm (Clause 26.4.2.1) for any type of exposure conditions.

5.12. Effective length of compression members [Table 28, IS 456]

Table 1. Effective length of compression members

Degree of end restraint of compression member	Symbol	Effective length
Effectively held in position and restrained against rotation in both ends		0.65 L
Effectively held in position @ both ends, restrained against rotation at 1 end		0.8 L
Effectively held in position at both ends, but not restrained against rotation		1L
Effectively held in position and restrained against rotation at one end, and at the other restrained against rotation but not held in position		1.2 L
Effectively held in position and restrained against rotation in one end, and at the other partially restrained against rotation but not held in position		1.5 L
Effectively held in position at one end but not restrained against rotation, and at the other end restrained against rotation but not held in position		2 L
Effectively held in position and restrained against rotation at one end but not held in position nor restrained against rotation at the other end		2 L

L – Unsupported length of the column

QUESTIONS

1. An RCC short column (with lateral ties) of rectangular cross section of 250 mm × 300 mm is reinforced with four numbers of 16 mm diameter longitudinal bars. The grades of steel and concrete are Fe415 and M20, respectively. Neglect eccentricity effect. Considering limit state of collapse in compression (IS 456: 2000), the axial load carrying capacity of the column (in kN, up to one decimal place), is [GATE 2018]

2. A column of size 450 mm × 600 mm has unsupported length of 3.0 m and is braced against side sway in both directions. According to IS 456: 2000, the minimum eccentricities (in mm) with respect to major and minor principal axes [GATE 2015]

A. 20 and 20

B. 26 and 21

C. 26 and 20

D. 21 and 15

3. A rectangular column 400 mm × 600 mm is reinforced with 0.8% reinforcement based on gross area. Fe 415 steel and M30 concrete are used. The ultimate load carrying capacity of the column is

A. 2136 kN

B. 2438 kN

C. 4320 kN

D. 3390 kN

4. The effective length of a column in building frames given in IS: 456- 2000 are based on

A. Wood's table

B. Wresler's table

C. Mohr's table

D. Bresler's table

5. A five meter long square RCC column is fixed at one end and hinged at the other end has minimum radius of gyration as 100 mm, its slenderness ratio is

A. 50 mm

B. 40 mm

C. 32.5 mm

D. 20 mm

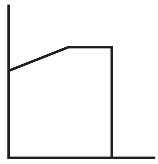
5. ***Answer:*** **B**

For a column with one end fixed and other end hinged, l_{eff} = 0.8 × unsupported length

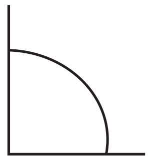
$$\lambda = \frac{l_{eff}}{r} = \frac{0.8 \times 5000}{100} = 40\text{mm}$$

6. A rectangular reinforced column (B × D) has been subjected to uniaxial bending moment M and axial load P. Characteristic strength of concrete is f_{ck}. Which among the following column design curves shows the relation between M (x-axis) and P (y-axis) quantitatively? (ESE-2006)

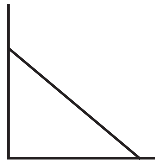
(A)



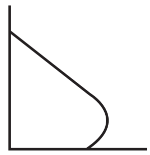
(B)



(C)



(D)



7. A column of 600 mm × 450 mm is having an unsupported length of 3 m, the design criteria of the column as per IS 456: 2000 will be

A. short along long and short dimensions

B. long along short and short along long dimensions

C. long along long and short dimensions

D. long along long and short along short dimensions

ANSWERS

1. ***Answer:*** **817**

$A_{sc} = 4 \times (\pi / 4) \times 16^2 = 804.25 \text{ mm}^2$

$A_c = (250 \times 300) - 804.25 = 74195.75 \text{ mm}^2$

$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$

$= (0.4 \times 20 \times 74195.75) + (0.67 \times 415 \times 804.25)$

$= 817.19$ kN

2. ***Answer:*** **B**

$$E_{min} = \frac{L}{500} + \frac{\text{Lateral dimension}}{30}$$

subject to a minimum of 20 mm.

Here, L = 3 m = 3000 mm

About major axis, $e_{min} = \frac{3000}{500} + \frac{600}{30}$

$= 26 \text{ mm} > 20 \text{ mm}$

About minor axis, $e_{min} = \frac{3000}{500} + \frac{450}{30}$

$= 21 \text{ mm} > 20 \text{ mm}$

∴ The minimum eccentricities are 26 mm and 21 mm.

3. ***Answer:*** **D**

$A_{sc} = (0.8/100) \times 400 \times 600 = 1920 \text{ mm}^2$

$A_c = 400 \times 600 - 1920 = 238080 \text{ mm}^2$

$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$

$= 0.4 \times 30 \times 238080 + 0.67 \times 415 \times 1920$

$= 3390.8$ kN

4. ***Answer:*** **A**

The effective length of compression members given in Table 28 of IS 456: 2000 is based on Wood's table.

5. ***Answer:*** **B**

For a column with one end fixed and other end hinged, I_{eff} = 0.8 × unsupported length

$$\frac{l_{eff}}{r} = \frac{0.8 \times 5000}{100} = 40 \text{ mm}$$

6. ***Answer:*** **D**

The interaction curves given in SP 16 resembles as in option'D'

7. ***Answer:*** **A**

L = 3000 mm

L/b = 3000/450 = 6.67 < 12

Hence, short column

L/d = 3000/600 = 5 <12

Hence, short column

EXERCISE

1. A structural member subjected to compression, has both translation and rotation restrained at one end, while only translation is restrained at the other end. As per IS 456: 2000, the effective length factor recommended for design is [GATE-2018]

A. 0.5

B. 0.65

C. 0.70

D. 0.80

2. A reinforced concrete column contains longitudinal steel equal to 1% of net cross-sectional area of the column.Assume modular ratio as 10. The loads carried (using the elastic theory) by the longitudinal steel and the net area of concrete are Ps and Pc respectivelyThe ratio Ps/Pc expressed as per cent is [GATE-2008]

A. 0.1

B. 1

C. 1.1

D. 10

3. A rectangular column section of 250mm × 400mm is reinforced with five steel bars of grade Fe-500 each of 20 mm diameter. Concrete mix is M-30Axial load in the column section with minimum eccentricity as per IS: 456-2000 using limit state method can be applied upto

[GATE-2005]

A. 1707.37

B. 1805.30

C. 1806.40

D. 1903.7

4. An RC short column with 300 mm × 300 mm square cross-section is made of M-20 grade concrete and has 4 numbers, 20 mm diameter longitudinal bars of Fe-415 steel. It is under the action of concentric axial compressive load. Ignoring the reduction in the area of due to steel bars, the ultimate axial load carrying capacity of the column is [GATE-2004]

A. 1659 kN

B. 1548 kN

C. 1198 kN

D. 1069 Kn

5. A reinforced concrete wall carrying vertical loads is generally designed as per recommendations given for columns. The ratio of minimum reinforcements in the vertical and horizontal directions is [GATE-1998]

A. 2:1

B. 5:3

C. 1: 1

D. 3:5

6. An R.C. short column, with 300 mm × 300 mm square cross section is made of M20 grade concrete and has 1numbers, 20 mm diameter longitudinal bars of Fe 415 steel. It is under the action of a concrete axial compressive load. Ignoring the reduction int he area of concrete due to steel bars, the ultimate axial load carrying capacity of the column is

A. 1659 kN

B. 1548 kN

C. 1198 kN

D. 1069 kN

7. Under the action of a concentric axial compressive load, a reinforced concrete short square column of size 300 mm is reinforced with 4 numbers of 25 mm diameter longitudinal bars of Fe415 steel. Concrete used is of grade M30. The ultimate axial load carrying capacity of the column is ____ kN.

A. 1800

B. 1200

C. 856

D. 1480

ANSWER KEY

1.	**D**	3.	**A**	5.	**D**	7.	**A**
2.	**D**	4.	**D**	6.	**D**		

Calculus of vector valued functions

A vector valued function in two dimensional space defined for $a \le t \le b$ can be expressed as $r = x(t)\,i + y(t)$, $a \le t \le b$ and in three dimensional space it is expressed as $\bar{r} = x(t)\,i + y(t)\,j + z(t)k$, $a \le t \le b$. Where x(t), y(t), z(t) are real functions of a real variable t. These fucntions are also called components of the vector valued function r(t).

A vector valued function $\bar{r}(t)$ defined at all points in some neighbourhood of a point t = a is said to be continuous at t = a if $\lim\limits_{t \to a} r(t) = r(a)$ ie, the limiting vector of the variable vector r(t) is the same as the vector r(a) as t approaches a. The vector function r(t) is continuous at each point t & T.

A vector valued function r(t) = x(t)i + y(t)j +z(t)k at t = a is continuous if and only if, the component functions x(t), y(t) and z(t) are continuous at t = a.

2.19 Derivative of vector valued functions

Let r(t) be a vector valued function defined at all points in a neighbourhood of a point t = a, if $\lim\limits_{t \to a} \dfrac{r(t) - r(a)}{t - a}$ exists and is denoted by $\dfrac{d}{dt} r(t)$ or $\acute{r}(a)$

2.20 Geometric interpretation of $\bar{r}(t)$

Let 'c' be the graph of a vector valued function r(t) is defined (exists) and non -zero for a given value of t. Let r(t) be computed at the terminal point of the radius vector r(t). Then r(t) is tangential to the curve 'c' and directed along the direction if increasing parameter 't'.

The vector funciton f(t) = x(t) i + y(t)j + z(t)k is differentiable at 't' if and only if the component functions x(t), y(t) are differentialbe at 't'. The derivative of r(t) = x(t)i + y(t) j + z(t)k is given by,

$$\frac{d}{dt} r(t) = \frac{d}{d(t)} x(t)i + \frac{d}{d(t)} y(t)j + \frac{d}{d(t)} z(t)k$$

2.21 Properties of vector functions

Let f(t) be a real valued function t, and $r_1(t)$ and $r_2(t)$ be vector valued functions of t and α and β be scalar. Then

1. $\dfrac{d\alpha}{dt} = 0$

2. $\dfrac{d}{dt}(\alpha r_1(t) + \beta r_2(t)) = \alpha \dfrac{d}{dt} r_1(t) + \beta \dfrac{d}{dt} r_2(t)$

3. $\frac{d}{dt}(f(t)r_1(t)) = f(t)\frac{d\,r_1(t)}{dt} + \frac{d}{dt}f(t)\,r_1(t)$

2.22 Derivatives of dot product and cross product of vector funciton

The following rules of differentiation are useful to differentiate combinations of vector valued functions

Let r(t) and s(t) be two vector valued functions of a scalar variable 't' then,

1. $\frac{d}{dt}(r(t)\,s(t)) = r(t)\frac{d}{dt}s(t) + \frac{d}{dt}r(t)\,.\,s(t)$

2. $\frac{d}{dt}(r(t) \times s(t)) = r(t)\frac{d}{dt}s(t) + \frac{d}{dt}r(t) \times s(t)$

2.23 Motion along a curve - velocity and acceleration

Let r(t) = x(t)i + y(t)j be the vector valued function describing position vector of a moving particle along a curve in a plane. Then the instantaneous rate of change of position, that is velocity of the particle is defined as $u(t) = \frac{dr(t)}{dt}$ and the instantanious acceleration of the particle is defined as $a(t) = \frac{du(t)}{dt} = \frac{d^2r(t)}{dt^2}$ and speed of the particle is given by $|u(t)| = \left|\frac{d}{dt}r(t)\right|$

Practice problems

Q 1. Find the velocity, speed, acceleration at the given time t of a particle moving along the given curve x = 1 + 3t, y = 2 - 4t, z = 7 + 2t, at t = 2 ?

Sol:

$r(t) = xi + yj + zk$

$= (1 + 3t)i + (2 - 4t)j + (7 + 2t)k$

$u(t) = \frac{dr(t)}{dt} = 3i - 4j + 2k$

velocity at t = 2, $u(t) = 3i - 4j + 2k$

acceleration = $r''(t) = 0$

speed = $|u| = \sqrt{9 + 16 + 4}$

$= \sqrt{29}$

Q 2. Find the position and velocity vectors of the particle given $a(t) = \frac{1}{(t+1)^2}j - e^{-2t}k$ $u(0) = 3i - j$ $r(0) = k$

$$u(t) = \int a(t)\, dt$$

$$= \int \frac{1}{(t+1)^2}\ j - e^{-2t}\, k\, dt$$

$$= \frac{-1}{1+t}\ j + \frac{e^{-2t}}{2} k + c_1$$

When t = 0,

$$u(0) = -j + \frac{k}{2} + c_1$$

$$\Rightarrow 3i - j = -j + \frac{k}{2} + c_1$$

$$c_1 = 3i - \frac{k}{2}$$

$$u(t) = \frac{-1}{(1+t)} j + \frac{e^{-2t}}{2}\ k + (3i - ½)^k$$

$$\int r(t)\, dt = \int \frac{-1}{1+t}\ j + \frac{e^{-2t}}{2}\ k + (3i - ½)k\Big]\, dt$$

$$= -\ln(t+1)j - \frac{e^{-2t}}{4} k + 3ti - \frac{tk}{2} + c_2$$

$$= 3ti - \mathit{l}n(t+1)j - \left(\frac{e^{-2t}}{4} + ½\, t\right) k + c_2$$

$$r(0) = k = \frac{-1}{4}\ k + c_2$$

$$c_2 = \frac{5}{4}\ k$$

$$r(t) = 3ti - \ln(t+1)j + \left(\frac{5}{4} - \frac{1}{4}\, e^{-2t} - \frac{1}{2}\, t\right) k$$

Q 3. Find the velocity and acceleration at t = 0, $r(t) = e^t i + e^{-2t} j + tk \quad t = 0$

$$r(t) = e^t i + e^{-2t} j + tk$$

$$v(t) = e^t i - 2e^{-2t} j + k$$

$$a(t) = e^t i + 4e^{-2t} j$$

u(t) at t = 0, $\quad v = i - 2j + k$

a(t) at t = 0, $\quad a = i + 4j$

2.24 Scalar Point Functions

At each point P of a region R we may associate a scalar denoted by $\phi(P)$. We then say that ϕ is a point-function over the region R.

2.25 Vector Point Functions

Each point P of a region R, there is associated a vector $\vec{F}(P)$, the function $\vec{F}$ is called a vector-point - function.

2.26 The Vector differential Operator ∇

The vector differential operator ∇(read as nabla) is defined as

$$\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z} \quad \text{where} \quad \vec{i}, \vec{j}, \vec{k}, \text{ are unit vectors}$$

The vector operator ∇ behaves like an ordinary vector, it satisfies all the properties of ordinary vectors.

2.27 Gradient (or slope of a scalar point function)

Let $\phi(x, y, z)$ be a scalar point functions and is continuously differentiable, then the vector

$$\text{grad } \phi - \nabla \phi = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)\phi = \vec{i}\frac{\partial \phi}{\partial x} + \vec{j}\frac{\partial \phi}{\partial y} + \vec{k}\frac{\partial \phi}{\partial z}$$

is called the gradient of the scalar funtion ϕ.

Note -1 *: Geometrically,* gradϕ *at a point is the normal vector to the surface as shown below.*

Note -2 *: Unit normal vector is given by* $\dfrac{\text{grad}\phi}{|\text{grad}\phi|}$

2.28 Properties of Gradient

1. If f and g are any two scalar point functions then $\nabla(f \pm g) = \nabla f \pm g$.
2. If f and g are two scalar point functions then grad (fg) = f grad g + g grad f
3. If f and g are two scalar point functions then $\nabla(f/g) = \dfrac{g\nabla f - f\nabla g}{g^2}$
4. Gradient of of constant is zero.

2.29 Directional Derivative

Let $f(x, y, z)$ be a differentiable function at (x_0, y_0, z_0) and $u = u_{1i} + u_{2j} + u_{3k}$ be a unit vector. Then the directional derivative in the direction of the unit vector $u = u_{1i} + u_{2j} + u_{3k}$ at (x_0, y_0, z_0) exists and is given by

$$D u\phi\,(x_0, y_0, z_0) = \frac{\partial \phi}{\partial x}u_1 + \frac{\partial \phi}{\partial y}u_2 + \frac{\partial \phi}{\partial z}u_3$$

$$= \nabla\phi\, u$$

Where the partial derivatives $\dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}, \dfrac{\partial f}{\partial z}$ are computed at (x_0, y_0, z_0) and $\bar{u}$ is a unit vector along which direction, we want to compute the directional derivation.

2.30 Properties of directional derivative

Let $f(x,y,z)$ be a function having continuous first order partial derivatives in some neghberhood at the point $f(x,y,z)$ then,

(i) If $\nabla f(x,y,z) = 0$, then directional derivatives along any direction at $p(x,y,z)$ vanishes.